Chapter 12 Probability

12.6 Binomial Distributions

Probability Distribution

- **Definition** - a function that gives the probability of each possible value of a random variable. The sum of all the probabilities in a probability distribution must equal 1.

Experiment: The sum when rolling two 6-sided dice.

x (sum)	2	3	4	5	6	7	8	9	10	11	12
Outcomes	1	2	3	4	5	6	5	4	3	2	1
P(x)	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

Random Variable x: A variable whose value is determined by the outcomes of a probability experiment.
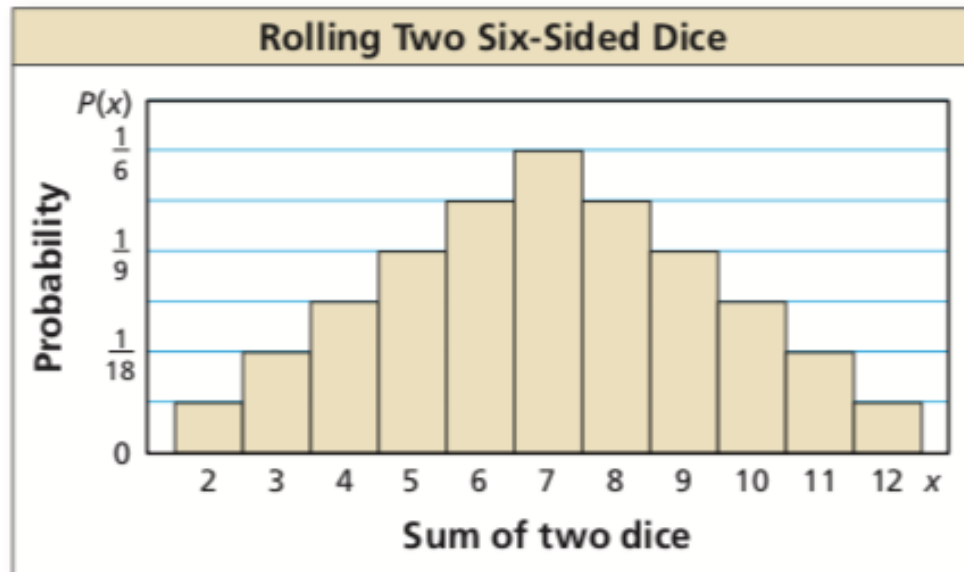
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Probability Distribution

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Histogram Representation of Distribution

Rolling Two Six-Sided Dice

a) What is the most likely sum?

b) What is the probability that the sum will be at least 10?

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Binomial Distribution

- **Definition** - The probabilities of the outcomes of a *binomial experiment*.

Binomial Experiment

- There are n independent trials.

- Each trial has only two possible outcomes: **success** and **failure**.

- The probability of success is the same for each trial. This probability is denoted by p. The probability of failure is 1 − p.

For a binomial experiment, the probability of exactly k successes in n trials is:

$$P(k \text{ successes}) = {}_n C_k \, p^k \left(1 - p\right)^{(n-k)}$$

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Constructing a Binomial Distribution

Experiment: According to a survey, about 33% of people ages 16 and older in the U.S. own an electronic book reading device, or e-reader. You ask 6 randomly chosen people (ages 16 and older) whether they own an e-reader. Draw a histogram of the binomial distribution for your survey.

Answer: The probability that a randomly selected person has an e-reader is p = 0.33. Because you survey 6 people, n = 6. You vary k from 0 to 6 people.
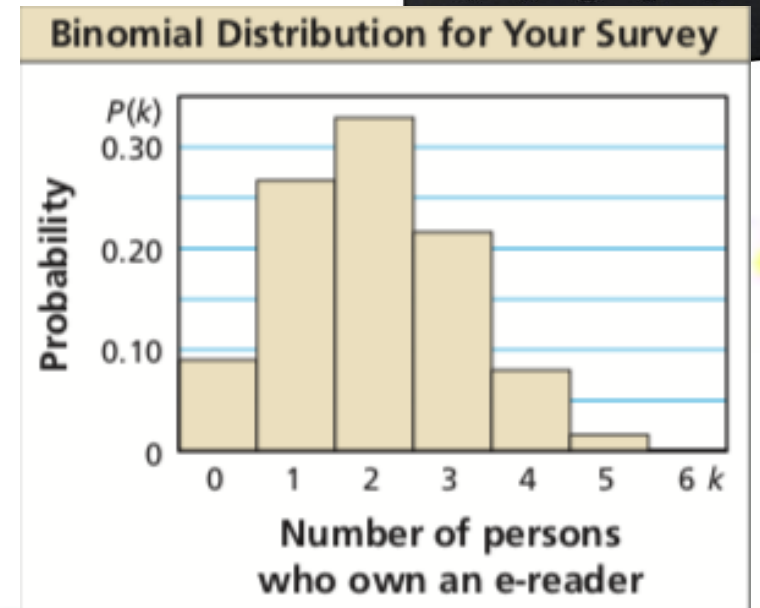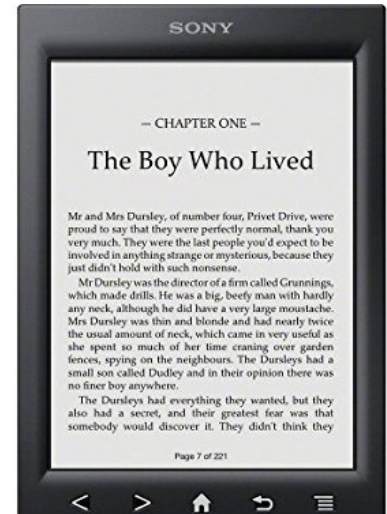
$$P(k = 0) = {_6}C_0(0.33)^0(0.67)^6 \approx 0.090$$

$$P(k = 1) = {_6}C_1(0.33)^1(0.67)^5 \approx 0.267$$
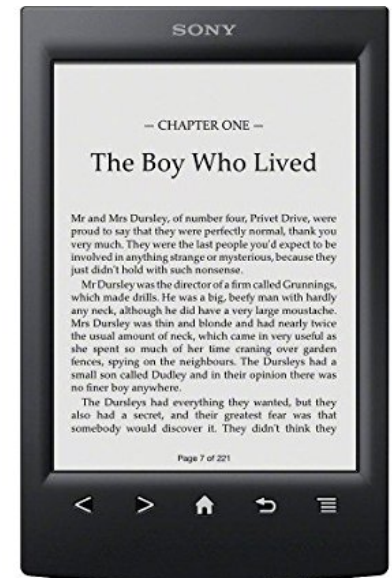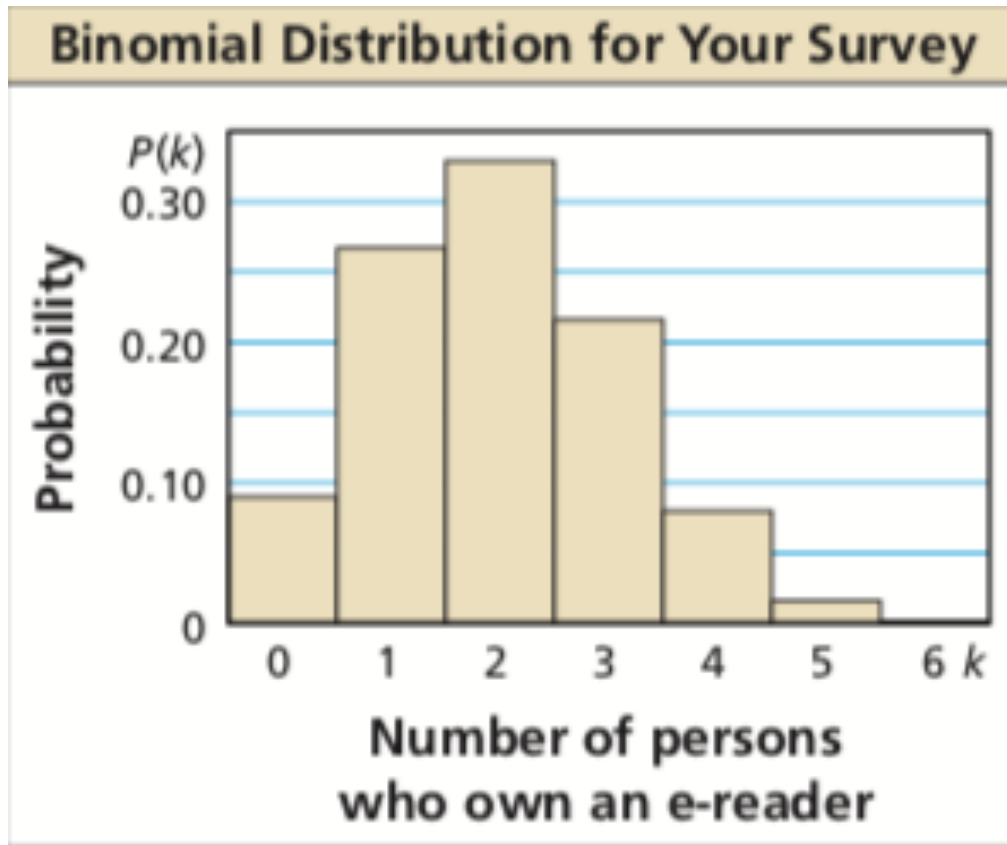
$$P(k = 2) = {_6}C_2(0.33)^2(0.67)^4 \approx 0.329$$

$$P(k = 3) = {_6}C_3(0.33)^3(0.67)^3 \approx 0.216$$

$$P(k \text{ successes}) = {_n}C_k\, p^k \left(1 - p\right)^{(n-k)}$$

Binomial Distribution for Your Survey

Number of persons who own an e-reader

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Constructing a Binomial Distribution

Binomial Distribution for Your Survey

Probability

P(k)
0.30
0.20
0.10
0

0 1 2 3 4 5 6 k

Number of persons
who own an e-reader

a) What is the most likely outcome of the survey?

b) What is the probability that at most 2 people own e-readers?

$$P(k\ successes) = {_n}C_k\, p^k \left(1 - p\right)^{(n-k)}$$